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A comparative study of soil slope using finite element and limit equilibrium methods to develop correlations for factor of safety

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ABSTRACT

This paper generally deals with the differences of results using limit equilibrium method and finite element methods in case of soil slope stability analysis to develop correlations for the value of factor of safety. Twenty different types of soil materials such as clay and clayey sand having different material properties are used in this research. Two softwares are used namely Slide and Phase2 which are limit equilibrium and finite element softwares respectively. A pre-defined slope is analyzed using both these methods. The factor of safeties are computed and provided in graphical form for both limit equilibrium and finite element methods used in this research where the difference of results in both these methods such as limit equilibrium and finite element can be easily compared and

checked. The main outcomes of this research are the correlations which are developed in between limit equilibrium and finite element methods. These correlations can be used for many different cases where the factor of safety for a soil slope is required.

1. INTRODUCTION

Since the construction of highways in hilly areas and the development of residential as well as commercial areas in mountainous regions, the importance of slope stability and overcoming land sliding has also got much attention of researchers. It is well understood that a huge economy is required for constructing highways in hilly areas but it is also risky in a sense that many times land sliding occurs and in result, human loss as well as economy loss is observed because of the slope failure. Another important issue is the construction of earthen dams for the purpose of storage of water as well as power production. The most important issue in earthen dams to be considered is the slope stability. If the slope is not stable enough to withstand against the driving and destabilizing forces, it will met with a failure and hence a huge economy as well as human losses may happen to occur. It is observed in histories that due to land sliding, human losses and economy losses are occurred all over the world.

Keeping the importance of this research area, many researchers worked in this research area and provided many techniques for making any soil slope stable whether it is earthen dam or a slope adjacent to a highway in hilly areas. Basically there are two methods normally used by researchers for analyzing soil slopes i.e. limit equilibrium method and finite element method. Although in many cases the results given by both these methods is not very different from each other but still it is observed that they have variation in results from each other.

In this paper both these methods such as Limit equilibrium and finite element are used to analyze a predefined soil slope and the variation in results is provided at the end. The main purpose of slope stability analysis is to make sure that the embankment, excavations, earth dams and landfills are safe and there is no chance of its sliding failure.

Research methodology

There are many different methods to analyze the slope stability which are briefly summarized by Duncan [6]. All of these methods are generally known as Limit Equilibrium Methods. These limit equilibrium methods include Bishop modified method, Janbu generalize procedure and force equilibrium method. Normally in all these methods, the soil mass is divided into a number of slices in which the direction of forces are being assumed which is the main distinguishing point among the various limit equilibrium methods as they have different assumptions regarding the forces direction.

The Limit Equilibrium approach has been to analyze slopes since 1930s. It depends on the type of problem whether it is circular slip surface or non-circular slip surface according to which the method is decided to be used. Also it depends on the accuracy of results required. In 1936, Fellenius method was the first limit equilibrium method which was adopted to analyze the circular slip surfaces [8]. It is also called Swedish circle method. The main assumption of this method is that the failure surface is assumed to be circular surface although other methods later on changed the assumptions to wedge shape and hence this method is now used very rarely.

In 1955, Bishop provided another limit equilibrium method to analyze the slopes which improved the accuracy of the results and hence it becomes basis for getting more accurate factor of safety values. In this method, an iterative procedure was used to analyze the slope and it was suited to computer methods where it is still in use [1]. Similarly there are many other limit equilibrium approaches by which any slope can be analyzed for the factor of safety value and to know how much the slope is safe and stable. Table 1 show all those methods along with their assumptions which are normally in use for slope stability analysis.

Table 1 Limit equilibrium methods

Method	Assumption
Ordinary or Fellenius	Inter-slice forces are neglected
Bishop's Simplified	Resultant inter-slice forces are horizontal (i.e., there are no inter-slice shear forces)
Janbu's Simplified	Resultant inter-slice forces are horizontal. An empirical correction factor is used

	to account for inter-slice shear forces.
Janbu's Rigorous	Location of the inter-slice normal force is defined by an assumed line of thrust.
Spencer	Resultant inter-slice forces are of constant slope throughout the sliding mass.
Morgenstern-Price	Direction of the resultant inter-slice forces is determined using an arbitrary function. The percentage of the function required to satisfy moment and force equilibrium is computed.
GLE	Direction of the resultant inter-slice forces is determined using an arbitrary function. The percentage of the function required to satisfy moment and force equilibrium is computed.
Corps of Engineers	Direction of the resultant inter-slice force is:
	(1) equal to the average slope from the beginning to the end of the slip surface or
	(2) parallel to the ground surface.
Lowe-Karafiath	Direction of the resultant inter-slice force is equal to the average of the ground surface and the slope at the base of each slice.

Another advanced method which is widely used for slope stability analysis is finite element method. This is also very old method and hence it is difficult to find the exact date of its invention but normally it was originated in 1940s after the work done by Courant [4] and Hrennikoff [12]. Many researchers provided a variety of finite element methods and there analysis methods are different from each other but mesh discretization is a common point between all of them in which they have discretized the whole body into small meshes known as elements. In 1960s and 1970s, the finite element method was much more explored by Ernest Hinton and Bruce Irons [11].

There are many finite element softwares currently in use such as Phase2, Midas, Abaqus, Plaxis, Ansys etc. For slope stability analysis, most of the finite element softwares use strength reduction method [7, 15, 18, 22]. In this method, there is no need of any assumptions for the failure surface [10]. The factor of safety is determined by using different shapes, sizes, loads, boundary conditions and material properties. In case of finite element method, a lot of efforts are required to find out the factor of safety as compare to limit equilibrium method. Especially in modelling and shaping the layout of the ground profile. Moreover additional data for getting the stress-strain behavior of the model is also required. Although nowadays the availability of finite element softwares made it much easy to analyze slopes but still comparative to limit equilibrium analysis, finite element analysis needs more time, efforts and input data. Whitman and Bailey were looking forward for such kind of softwares which could manage to model and analyze the slopes for finding out the factor of safety using finite element analysis [21]. Moreover Chowdhury suggested to develop some other alternative to limit equilibrium method [3], while commenting on Sarma [17]. Such kind of softwares is available such as Phase2, Plaxis and Flac etc. In many of these softwares, both Mohr-Coulomb and Hoek-Brown methods could be used. One of this finite element software is Phase2 manufactured by rocscience. In this paper, Phase2 software is used which is a finite element two dimensional software.

The finite element method can be used to compute displacements and stresses caused by applied loads. For calculating the overall factor of safety, the computed stresses needs to be additionally processed. The construction pore water pressure and displacements can be estimated using finite element method. This method could be used for field control of construction as well as if the damage to adjacent structures is to be considered. In case of the variation of displacements and pore water pressure values in field and computed values, the difference could also be investigated using finite element method. It also provides pattern of displacements by which the complex failure mechanism can be calculated.

In case of limit equilibrium method, the factor of safety value depends on locating the potential slip surface which is the most critical surface plane. Therefore in complex conditions, it is not easy to know the failure modes in advance, especially if reinforcement or the structural members i.e. geotextiles, concrete retaining walls or sheet piles are included. Once a potential failure mechanism is identified, the factor of safety against a shear failure developing by that mode can be computed using limit equilibrium procedures. A finite element analysis gives the estimates of mobilized stresses and forces. The finite element method may be particularly useful in judging what strengths should be used when materials have very dissimilar stress-strain and strength properties, i.e., where the strain compatibility is to be considered. The finite element method can also help to identify the local regions where overstress is occurred and it causes cracking in brittle materials. Also, the finite element method is helpful in finding

out how reinforcement will respond in embankments. Finite element analyses may be useful in areas where new types of reinforcement are being used. An important input to the stability analyses for reinforced slopes is the strength in the reinforcement. The finite element method can also provide guidance for developing the force that will be used. The elastic plastic finite element method has been shown to be useful alternative to conventional slope stability analysis techniques [19, 24, 9].

If desired, factors of safety equivalent to those computed using limit equilibrium analyses can be computed from results of finite element analyses. The complete procedure for using the finite element method to calculate the factors of safety are as follows:

(1) Perform an analysis using the finite element method to find the stresses in the slope.

(2) Select a trial slip surface.

(3) Subdivide the slip surface into small segments.

(4) Find out the normal stresses as well as shear stresses along the assumed slip surface. This will require interpolation of values of stress from the values computed at Gauss points in the finite element mesh to obtain the values at selected points on the slip surface. If an effective stress analysis is performed, subtract the pore pressure to determine the effective normal stresses on the slip surface. The pore pressures are determined from the same finite element analysis if a coupled analysis is performed to calculate the stresses and deformations. The pore pressures can also be determined from a separate steady seepage analysis if an uncoupled analysis was performed to compute stresses and deformations.

(5) Use the normal stress and the shear strength parameters, c and φ, or c' and φ', to compute the shear strength at points along the shear surface. Use total normal stresses and total stress shear strength parameters for total stress analysis and effective normal stresses and effective stress shear strength parameters for effective stress analyses.

(6) Compute an overall factor of safety using the following equation:

$$F = \frac{\Sigma s_i \Delta l}{\Sigma \tau_i \Delta l}$$

(1)

Where:

si = available shear strength computed in step (4)

Ti = shear stress computed in step (3)

Δl = length of each individual segment into which the slip surface has been subdivided

Studies have shown that factors of safety determined using the procedure described above are, for practical purposes and mostly it is equal to factors of safety determined using accurate limit equilibrium methods.

Local, such as point by point factors of safety can also be computed using the stresses and shear strength properties at selected points in a slope. Some of the local factors of safety will be less than the overall minimum factor of safety computed from the equation or limit equilibrium analyses. Local factors of safety of one or less do not necessarily indicate that a slope is unstable. Stresses will be redistributed from points of local failure to other points where the local factor of safety is greater than 1. As long as the overall factor of safety is greater than 1, the slope will be safe and stable. Where estimates of movements as well as factor of safety are required to achieve design objectives, the effort required to perform finite element analyses can be justified.

2. MATERIAL AND METHODS

Local soil properties were used and same material properties were used in case of limit equilibrium analysis and finite element analysis. Cohesion (c), friction (φ) and unit weight (γ) ranges are taken from different experimental analysis for clay and clayey sand. Basically two types of soil from different sites were taken in this analysis that is clay and clayey sand. In case of clay, the cohesion stress was in the range of 10 to 20 KN/m2 while in clayey sand, it was very low such as from 1 to 10 KN/m2. Phi (φ) value for the same soil was in range of 27° to 37°. While for clayey sand, it was in range 16 to 18 KN/m3. The unit weight for clay was in the range of 13 to 19 KN/m3. Table 2 shows the material properties such as values of cohesion, friction and unit weight used in this analysis for total twenty number of analysis. And *table 3 shows the relationship table between relative density and angle of friction in case of cohesionless soil [5].*

Table 2 Material properties

Material Number	Cohesion	Friction	Unit Weight (KN/m³)	Material Type
	(KPa)	(φ)		
1	10	27	13	Clay
2	11	28	13.6	Clay
3	12	29	14.2	Clay
4	13	30	14.8	Clay
5	14	31	15.4	Clay
6	15	32	16	Clay
7	16	33	16.6	Clay
8	17	34	17.2	Clay
9	18	35	17.8	Clay
10	19	36	18.4	Clay
11	1	25	16.366	Clayey Sand
12	2	26	16.464	Clayey Sand
13	3	27	16.562	Clayey Sand
14	4	28	16.758	Clayey Sand
15	5	29	16.856	Clayey Sand
16	6	30	16.954	Clayey Sand
17	7	31	17.052	Clayey Sand
18	8	32	17.15	Clayey Sand
19	9	33	17.248	Clayey Sand
20	10	34	17.346	Clayey Sand

Table 3 Relationship between Relative density and Angle of friction of cohesionless soil

State of Packing	Relative Density (%)	Angle of friction, (φ') (deg.)
Very Loose	<20	<30
Loose	20-40	30-35
Compact	40-60	35-40
Dense	60-80	40-45
Very Dense	>80	>45

The unit weight of clayey sand was in the range of 16.366 to 17.346 KN/m³. There are different types of cohesionless soil such as medium coarse sand, fine grained, very coarse etc. In this paper, medium coarse sand is taken in the analysis. The values of unit weight according to the friction angle are mentioned in Table 4.

Table 4 Unit weight and Phi values for clayey sand

Phi, φ	γ, Unit weight (g/cm^3)	γ, Unit weight (KN/cm^3)
25	1.67	16.366
26	1.68	16.464
27	1.69	16.562

28	1.71	16.758
29	1.72	16.856
30	1.73	16.954
31	1.74	17.052
32	1.75	17.15
33	1.76	17.248
34	1.77	17.346

The factor of safety is varying in different conditions having different soil properties. For example if the cohesive forces among the soil particles are high, then the FS will also be high. Similarly if the friction angle between the soil particles is low then the factor of safety will also having a low value and vice versa.

Mohr-Coulomb method

The slope model which is used in this analysis is much similar to the model used by Zhang [22] and has been used by various researchers as a part of the validation of the particular 3D slope stability methods [14, 13, 2].

In geotechnical engineering, mostly the Mohr-Coulomb criteria is used to define the shear strength of soils and rocks at various effective stresses, therefore in this paper, the same criteria is used throughout the analysis. The relation between Mohr-Coulomb failure criterions is linear and it is always obtained from the graph between shear strength versus normal stress. The relation is shown below:

$$T = c + \delta \tan \phi \tag{2}$$

Where T is the shear strength, δ is the Normal stress, c is cohesion and ϕ is frictional angle

Modeling and analysis using Phase2 software

Phase2 is very useful two dimensional finite element software which is used for soil and rock slope stability analysis. Phase2 can be used for many engineering projects which include excavation design, slope stability analysis, groundwater seepage analysis, probabilistic analysis, consolidation, and dynamic analysis etc. A free three dimensional finite element slope stability analysis program is described in detail by Smith & Griffiths [19].

Using Phase 2, complex and multi-stage slope models can be easily modelled and quickly analyzed. Tunnels in weak or jointed / fractured rock, underground powerhouse caverns, open pit mines and slopes, embankments, earth structures etc. Timely failure, soil support interaction and a variety of other problems can be analyzed.

3. RESULTS AND DISCUSSION

The pre-defined soil slope was analyzed using three ways.

Natural form

Stepping form

Nailed form

The differences in the results are shown in three steps.

Analysis using finite element software (Phase 2)

Same model which was analyzed using both limit equilibrium analysis as well as finite element analysis. Figure 1 shows that factor of safety value achieved by finite element method in case of natural slope condition which is 0.92. The values in the left table show the yielding percentage.
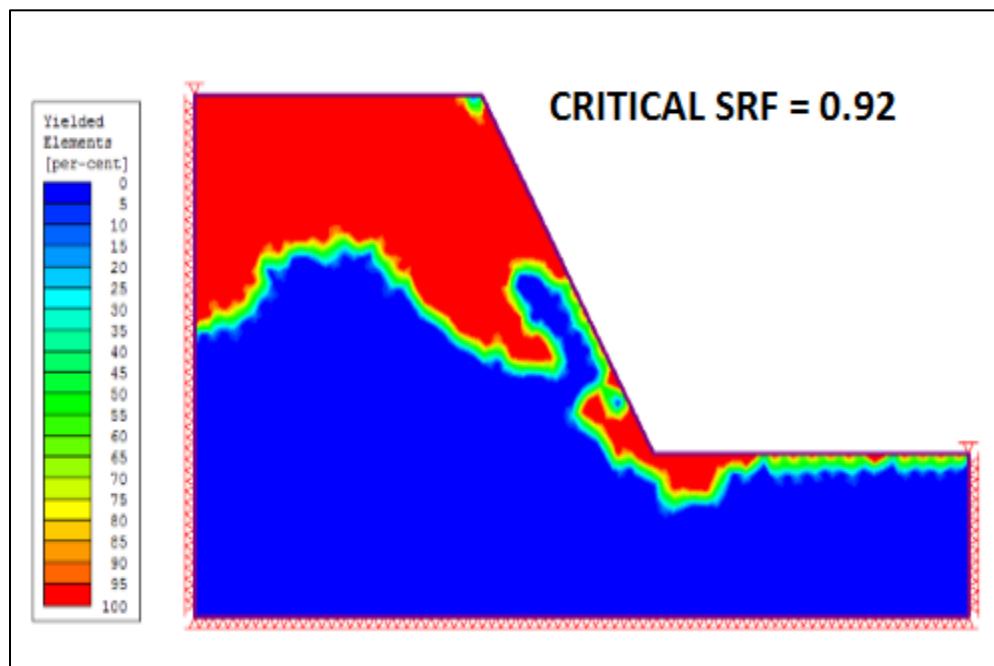
Figure 1 Factor of safety in case of natural condition for material 1
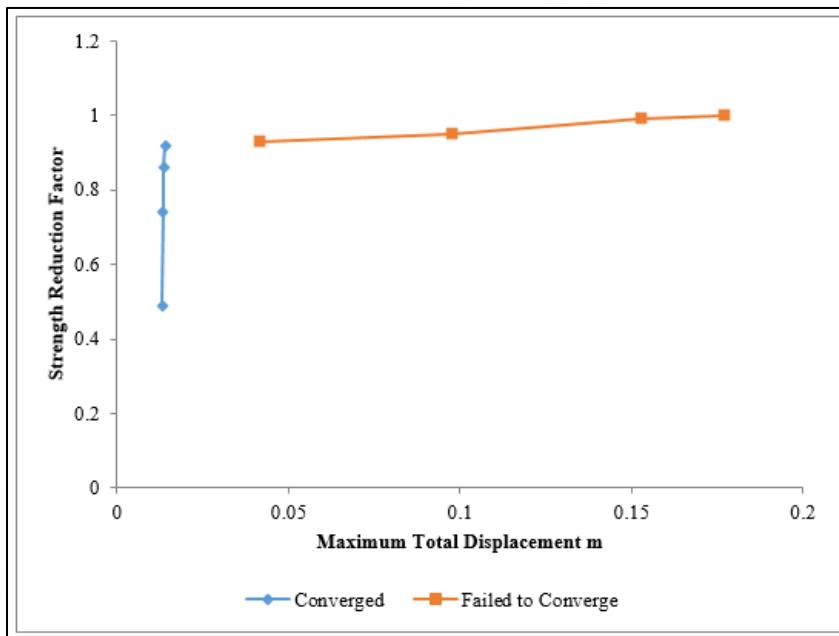

Figure 2 Graph between strength reduction factor and maximum displacement for material 1

Figure 2 shows the graph between strength reduction factor and maximum displacement for material 1. There were twenty different types of material used in case of limit equilibrium method which are mentioned in table 1. For simplification purpose in this paper, five out of twenty materials are taken for plotting the comparison graphs between limit and finite element methods. Those five material properties are material 1, material 5, material 10, material 15 and material 20. The slope was analyzed in five conditions such as natural form, stepped form, nailed 3m, 5m and 8m. This finite element 2D analysis was done in three steps such as in natural condition, and then stepping and then nailing is done. Table 5 shows the factor of safety values in case of finite element methods for all the five material types along with the details mentioned regarding their condition such as natural, stepped or nailed.

Table 5 Factor of safety values in case of finite element method

Condition	Material 1	Material 5	Material 10	Material 15	Material 20
Natural	0.92	1.09	1.28	0.63	0.94
Stepped	1.08	1.27	1.5	0.74	1.12
Nailed (3m)	1.08	1.28	1.51	0.81	1.15
Nailed (5m)	1.19	1.41	1.66	0.82	1.23
Nailed (8m)	1.2	1.42	1.67	0.84	1.23

Table 5 clearly shows the variation in the values of factor of safeties i.e. in natural form it is less compare to stepped form and stepped form gives less factor of safety compare to nailed form. Moreover with the increase of nail length, the factor of safety is increased which can be seen in table 5. Experience shows that in case of clayey sand, there is no much effect of nail length on the factor of safety value while it gives very better results in case of pure clay. Table 6 shows the factor of safety values for the same slope using limit equilibrium method.

Table 6 Factor of safety values in case of limit equilibrium method

Condition	Material 1	Material 5	Material 10	Material 15	Material 20
Natural	0.953	1.125	1.317	0.66	0.973
Stepped	1.097	1.296	1.522	0.809	1.152
Nailed (3m)	1.091	1.288	1.515	0.90	1.169
Nailed (5m)	1.325	1.563	1.808	1.128	1.484
Nailed (8m)	1.545	1.799	2.017	1.169	1.586
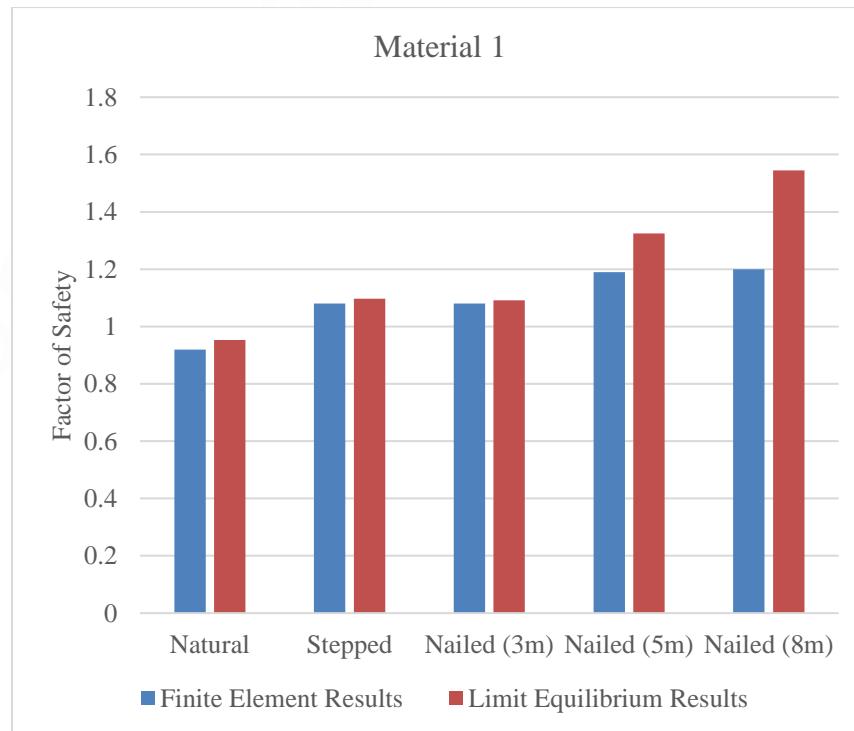
Figure 3 Results comparison for material 1

Results comparison of limit equilibrium and finite element methods

Figures 3 to 7 shows the difference of results between limit equilibrium method and finite element method for different slope conditions and by variation of nail length etc. which are mentioned in each figure.

Figure 3 clearly shows the variation of factor of safety value in both limit and finite element methods. In case of natural, stepped and 3m nailed condition, there is very less variation in the values of factor of safety values while in case of 5m and 8m nailed length, the variation is higher. It shows that increasing the length of anchored nail above 4m gives better values but note that the 5m and 8m nails are for material 15 and 20 which are basically clayey sand. Experience shows that this is not true in case of pure clay as the results are almost symmetric. Similarly all other graphs are shown here in which the same criterion is considered.
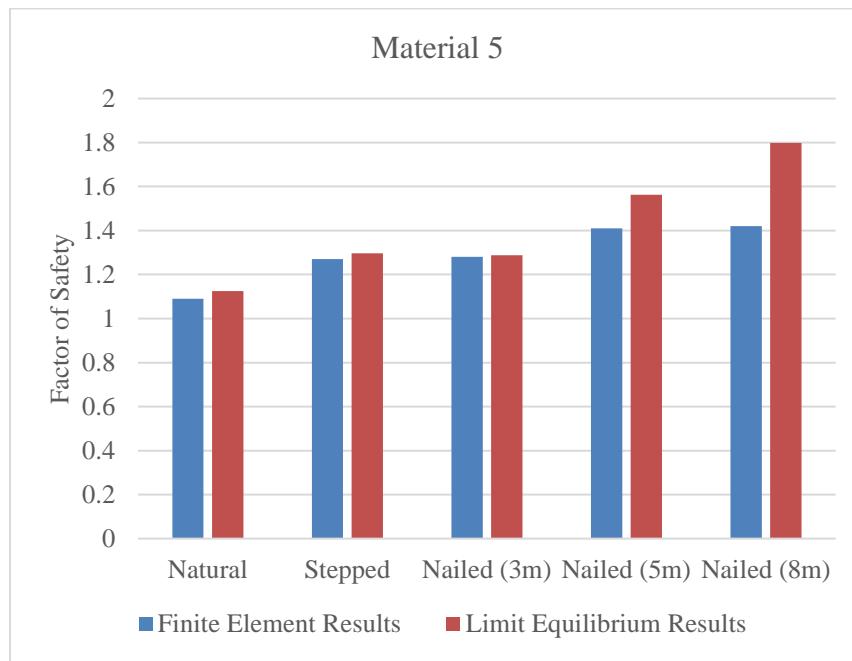
Figure 4 Results comparison for material 5
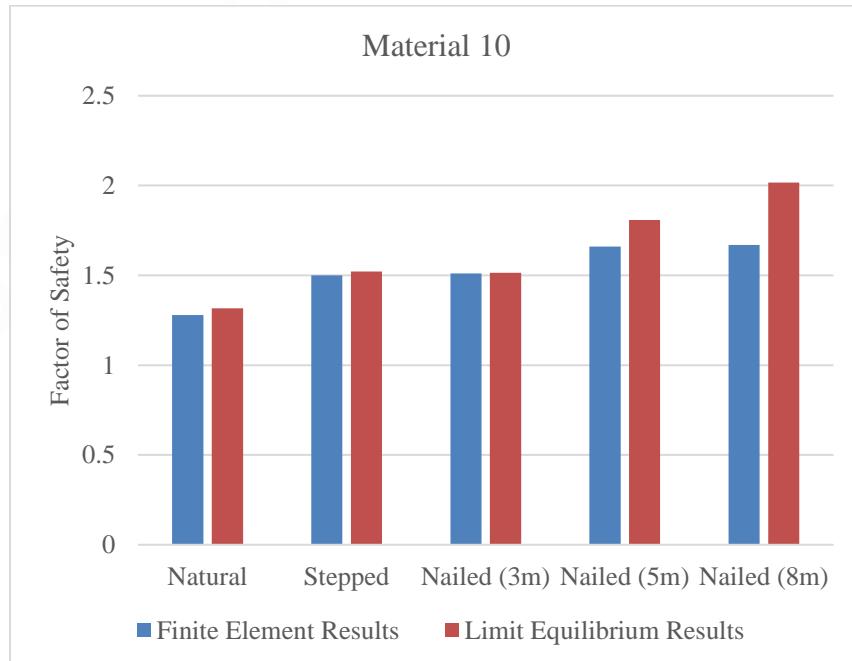
Figure 5 Results comparison for material 10

In all this analysis, two types of material having different values of cohesion, friction and unit weight are used. The figures from 3 to 7 are for material 1, 5, 10, 15 and 20. It is clear from table 1 that material 1, 5 and 10 are clay and 15, 20 are clayey sand. Therefore the factors of safeties are giving different variations compare to each other. It is also noted that nail length is also having great impact on the safety value.
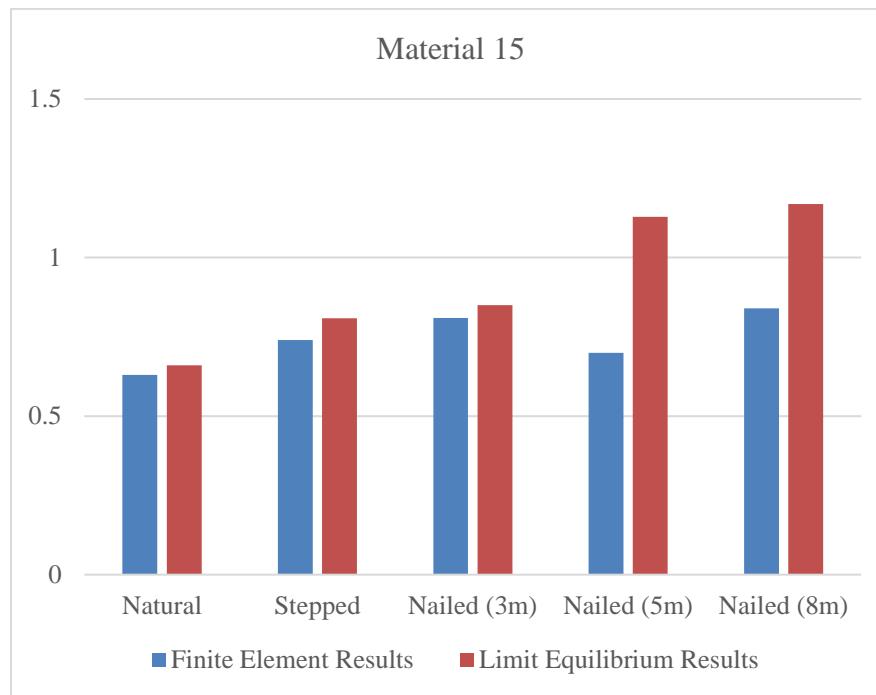

Figure 6 Results comparison for material 15
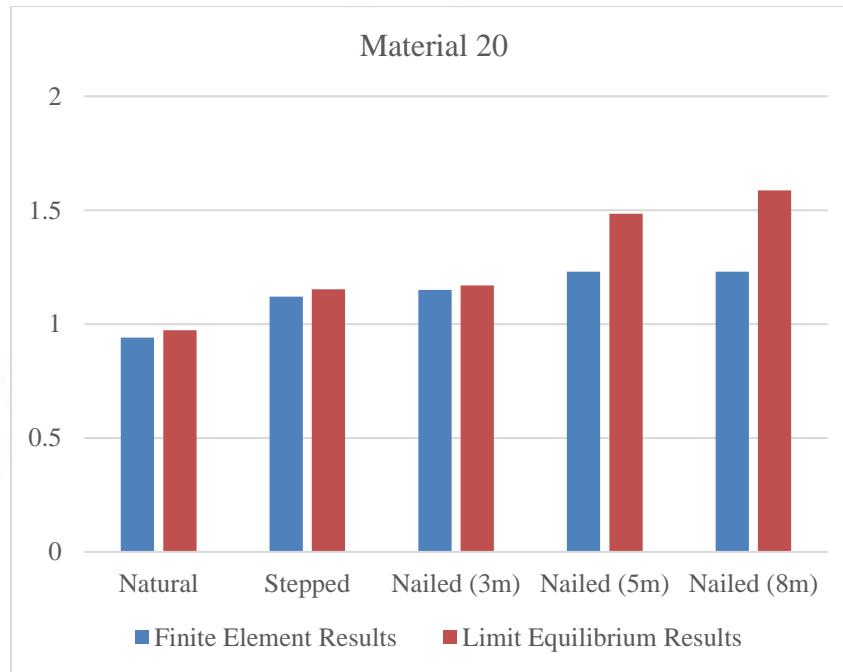
Figure 7 Results comparison for material 20

Correlations equations developed from this analysis

The graphs in figure 8 show the relation of factor of safety value between limit equilibrium and finite element methods in all the three cases such as natural stepped and nailed conditions. While table 7 shows the correlation equations for the five cases.
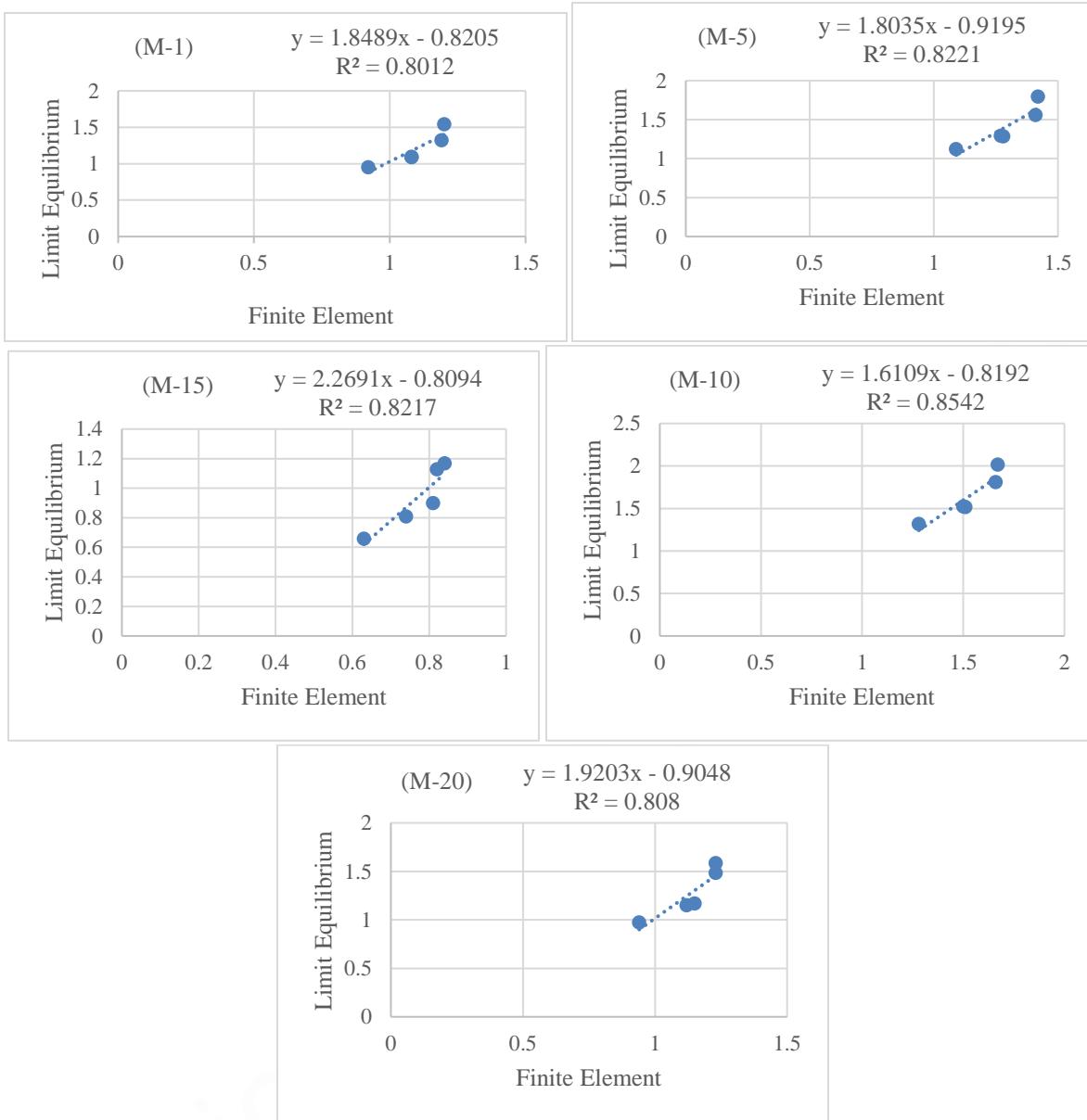
Figure 8 Factor of safety comparison graphs between limit equilibrium and finite element methods for material 1, 5, 10, 15 and 20 respectively.

A mean value for all these five cases will give the final correlation equation as below:

$$LE = 1.89 \times FE - 0.86 \text{ and } R^2 = 82\% \tag{3}$$

Where;
LE = Limit Equilibrium
FE = Finite Element
R square shows the equation applicability percentage.
Hence we can write this equation in case of factor of safety as:

$$(FS)_{LE} = 1.89 * (FS)_{FE} - 0.86 \tag{4}$$

This correlation equation of factor of safety can be used for other soil types as well.

4. CONCLUSIONS

From all the above studies, the following conclusions were drawn:

- The correlation developed between limit equilibrium and finite element method is very useful and can be used in other soil types to know the accurate value of the slope safety.
- If any slope stability design includes nailing, it is suggested to use finite element analysis as it gives better and economical results compare to limit equilibrium analysis.
- Although limit equilibrium methods are old methods but their results are almost similar to finite element methods in case of noncomplex geometries.
- Finite element method is giving better and improved results in case of complex geometries as the stresses and strains are not symmetric.
- In case of nailed slope where the nail length is more than 4m, the factor of safety values between limit equilibrium and finite element method have more variation depends on the nail strength properties.

A large number of water-retaining earthen dams were affected by the earthquake [16]. Therefore it is recommended that seismic analysis of the same work can be done to know the variation and differences in results. Adding pore water pressure and different layouts may also give different results. Moreover different nail types could be used to know how much variation is there in the factor of safety by changing the nail properties.

Conflicts of Interest: The authors declare no conflict of interest.

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